

27.10

The net force exerted on the electron (of charge  $q_e$  moving with velocity  $\vec{\mathbf{v}}$ ) is  $\vec{\mathbf{F}} = q_e(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$ . For the electron to undergo no acceleration  $\vec{\mathbf{F}}=0$ , and so  $\vec{\mathbf{E}}+\vec{\mathbf{v}}\times\vec{\mathbf{B}}=0$ , which requires  $E = |\vec{\mathbf{v}} \times \vec{\mathbf{B}}| = vB \sin \theta$ . Plug in  $E = 1000 \,\mathrm{V/m}$ ,  $B = 20.0 \times 10^{-2} \,\mathrm{T}$ , and  $\theta = 90.0^{\circ}$  to obtain

$$v = \frac{E}{B\sin\theta} = \frac{1000 \,\text{V/m}}{(20.0 \times 10^{-2} \,\text{T})(\sin 90.0^{\circ})} = 5.00 \times 10^{3} \,\text{m/s}.$$

27.15

An electron in the beam enters the crossing electric and magnetic field with  $v_x = E/B$  [see Chapter 19). The time it takes for the electron to pass through the E-field in between the parallel plates of length L is  $t=L/v_x$ , during which it is being accelerated in the y-direction by the electric field at the rate of of  $a_y = F_{\rm E}/m_{\rm e} = eE/m_{\rm e}$ . So as it emerges from the E-field the x-component of its velocity remains  $v_x$ , while the y-component increases from zero to

$$v_{_{y}}=a_{_{y}}t=\left(\frac{eE}{m_{_{e}}}\right)\left(\frac{L}{v_{_{x}}}\right)\;.$$

Thus  $\tan\theta = v_y/v_x = (eE/m_e)(L/v_x)/v_x$ , or  $e/m_e = v_x^2 \tan\theta/EL$ . Now, If  $v_x \gg v_y$ . then  $\tan \theta \approx \theta$  (as  $\theta \ll 1$ ), whereupon

$$\frac{e}{m_{\rm e}} = \frac{v_x^2 \tan \theta}{EL} \approx \frac{v_x^2 \theta}{EL} = \frac{(E/B)^2 \theta}{EL} = \frac{E\theta}{B^2 L} \,,$$

where we used  $v_x = E/B$  again.

From the hint we know that we must use relativistic approach. The KE of the electron as it emerges from the accelerating potential V is KE = eV, which is the difference between its rest energy  $E_o = m_e c^2$  and the total energy  $E = \gamma m_e c^2$ , where  $\gamma = 1/\sqrt{1 - v^2/c^2}$ , with v its final

$$\mathrm{KE} = eV = \mathrm{E} - \mathrm{E}_{\mathrm{o}} = (\gamma - 1) m_{\mathrm{o}} \mathrm{c}^2 \,.$$

Solve for  $\gamma$ :

$$\gamma = \frac{eV}{m_{\rm e}{\rm c}^2} + 1 = \frac{e \cdot 1.00 \times 10^6 \, {\rm V}}{0.510 \, 9 \times 10^6 \, {\rm eV}} + 1 = 2.9573 \, ,$$

where we noted that  $1 e \cdot V \equiv 1 eV$ . Thus

$$v = c\sqrt{1 - \frac{1}{\gamma^2}} = (2.998 \times 10^8 \,\mathrm{m/s})\sqrt{1 + \frac{1}{(2.957 \,3)^2}} = 2.82 \times 10^8 \,\mathrm{m/s}.$$

X-ray diffraction by crystals satisfy the Bragg equation, Eq. (27.3):  $2d\sin\theta_{\rm m}=m\lambda$ . Here d is the spacing between adjacent atomic planes,  $\theta_{\rm m}$  is the angle at which the m-th order reflection of the beam occurs, and  $\lambda$  is the wavelength of the incident beam. The first-order reflection angle  $\theta_1$  then satisfies

$$\sin \theta_1 = \frac{m\lambda}{2d} \bigg|_{m=1} = \frac{\lambda}{2d} \,.$$

Plug in  $d=0.303\,\mathrm{nm}$  and  $\lambda=0.090\,\mathrm{nm},$  and solve for  $\theta_1$ :

$$\theta_1 = \sin^{-1} \frac{\lambda}{2d} = \sin^{-1} \left[ \frac{0.090 \,\mathrm{nm}}{2(0.303 \,\mathrm{nm})} \right] = 8.5^{\circ}$$