Solutions to Problems

23.1

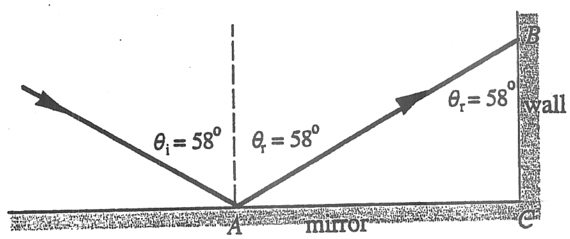
From the discussion in the text we know that the amount of scattering is proportional to $(1/\lambda)^4$, where λ is the wavelength of the light being scattered. Thus the ratio in question is

$$\frac{(1/\lambda_r)^4}{(1/\lambda_v)^4} = \left(\frac{\lambda_v}{\lambda_r}\right)^4 = \left(\frac{390\ \text{nm}}{780\ \text{nm}}\right)^4 = 0.0625 = 6.25\%\,.$$

23.8

Refer to the figure to the right. The angles of incidence and reflection are both 58°. Look at the right-angled triangle $\triangle ABC$. We are given $\overline{AB} = 5.0\,\text{m}$ and wish to find $\overline{AC}$, which satisfies $\overline{AC}/\overline{AB} = \sin\theta_r$. Solve for $\overline{AC}$:

$$\overline{AC} = \overline{AB}\sin\theta_r = (5.0\,\text{m})(\sin 58°) = 4.2\,\text{m}\,.$$



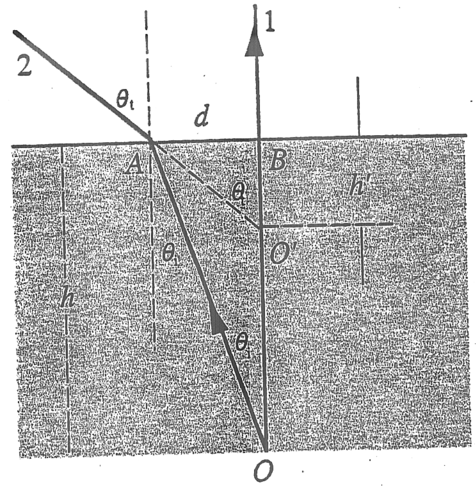
23.41

Apply the Law of Refraction, Eq. (23.3), which relates the angle of incidence θ_i in a medium having index of refraction n_i with the angle of transmission θ_t in a medium having index of transmission n_t: $n_i \sin\theta_i = n_t \sin\theta_t$. Plug in $\theta_i = 55°$, $\theta_t = 40°$, and $n_i = 1.00$ (for air) to find n_t, the index of refraction of the liquid:

$$n_t = \frac{n_i \sin\theta_i}{\sin\theta_t} = \frac{(1.00)(\sin 40°)}{\sin 55°} = 1.3\,.$$

23.45

Consider an arbitrary point O, a distance h below the surface of the water. Imagine two light rays, labeled 1 and 2, respectively, that emerge from O into the air. Ray 1 is perpendicular to the air-water boundary and makes its way into the air undeflected, while ray 2 is incident upon the boundary at an angle θ_i and emerges into the air at an angle θ_t, as shown. Viewed from above in the air the apparent position of point O is O', which appears to be the location where rays 1 and 2 would originate had they not been deflected. Thus the apparent depth of point O is h', which is obviously shallower than h, the true depth.



To find h' in terms of h, apply the Law of Refraction, $n_i \sin\theta_i = n_t \sin\theta_t$, where in this case $n_i = 1.333$ for water and $n_t = 1.00$ for air, so $\sin\theta_i = (n_t/n_i)\sin\theta_t = (1.00/1.333)\sin\theta_t = \frac{3}{4}\sin\theta_t$. If we view the water almost perpendicularly then both θ_i and θ_t are very small, in which case $\sin\theta_i \approx \tan\theta_i$ and $\sin\theta_i \approx \tan\theta_i$. Thus θ_i is related to θ_t by

$$\tan\theta_i \approx \frac{3}{4}\tan\theta_t\,.$$

Now consider the right-angled triangles $\triangle OAB$ and $\triangle O'AB$, in which $d/h' = \tan\theta_t$ and $d/h = \tan\theta_i$. Solve for h' and h to obtain $h' = d/\tan\theta_t$ and $h = d/\tan\theta_i$. Divide these two equations to obtain

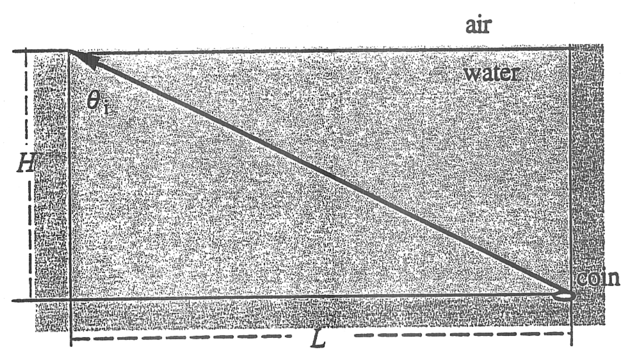
$$\frac{h'}{h} = \frac{d/\tan\theta_t}{d/\tan\theta_i} = \frac{\tan\theta_i}{\tan\theta_t} = \frac{3}{4}\,,$$

where we used the relation between $\tan\theta_i$ and $\tan\theta_t$ we derived earlier. This proves that $h' = \frac{3}{4}h$.

23.46

Consider a light ray from the coin. It approaches the water-air boundary at an angle of incidence θ_i which satisfies $\tan\theta_i = L/H$ (see the figure to the right), where $H = 3.00\,\text{m}$ is the depth of the pool and $L = 4.00\,\text{m}$ is its length. Thus

$$\theta_i = \tan^{-1}\frac{L}{H} = \tan^{-1}\frac{4.00\,\text{m}}{3.00\,\text{m}} = 53.1^\circ\,.$$



This angle, however, has exceeded the critical angle for total internal reflection on the water-air boundary, $\theta_c = \sin^{-1}(n_{\text{air}}/n_{\text{water}}) = \sin^{-1}(1.00/1.333) = 48.6^\circ$ [see Eq. (23.5) for θ_c]. Thus the light from the coin is reflected back into the water and the coin is therefore invisible to the observer.

23.49

Look at Fig. P49. Light is incident on the air-water boundary at an angle of θ_i which satisfies $\tan\theta_i = 15\,\text{cm}/20\,\text{cm} = 3/4$. θ_t is in a 3-4-5 right-angled triangle, so $\sin\theta_t = 3/5$. Also, the angle of transmission into the air is $\theta_t = 90° - 20° = 70°$. The index of refraction n_i for the liquid can then be found from Eq. (23.3), $n_i \sin\theta_i = n_t \sin\theta_t$:

$$n_i = \frac{n_t \sin\theta_t}{\sin\theta_i} = \frac{(1.00)(\sin 70°)}{3/5} = 1.6\,,$$

where we noted $n_t = 1.00$ for air.

23.58

The problem asks for the critical angle θ_c for total internal reflection to occur on the boundary between water and air. θ_c satisfies Eq. (23.5), $\sin\theta_c = n_t/n_i$, where $n_t = 1.000$ (for air) and $n_i = 1.333$ (for water). Thus the minimum angle in question is

$$\theta_c = \sin^{-1}\frac{n_t}{n_i} = \sin^{-1}\frac{1.000}{1.333} = 48.61°\,.$$

23.61

Denote the indices of refraction for the benzene and the gas as n_b and n_g, respectively. According to the problem statement the critical angle for total internal refraction on the benzene-gas boundary is $\theta_c = 45.0°$. Plug in this value of θ_c, along with $n_b = 1.501$, into Eq. (23.5), $\sin\theta_c = n_t/n_i = n_g/n_b$, and solve for n_g:

$$n_g = n_b \sin\theta_c = 1.501(\sin 45.0°) = 1.06\,.$$

24.1

First, use the Lensmaker's Formula, Eq. (24.4), to find f:

$$f = \left[(n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]^{-1} = \left[(1.50 - 1)\left(\frac{1}{0.50\,\text{m}} - \frac{1}{-0.50\,\text{m}}\right)\right]^{-1} = 0.50\,\text{m}\,.$$

Now locate the image from Eq. (24.6), $1/s_o + 1/s_i = 1/f$, or

$$s_i = \frac{s_o f}{s_o - f} = \frac{(1.00\,\text{m})(0.50\,\text{m})}{1.00\,\text{m} - 0.50\,\text{m}} = 1.0\,\text{m}\,,$$

i.e., the image is located 1.0 m to the right of the lens.

24.5

Use Eq. (24.4) for the focal length f, with $n_l = 1.58$, $R_1 = +1.00\,\text{m}$, and $R_2 = -1.00\,\text{m}$:

$$f = \left[(n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]^{-1} = \left[(1.58 - 1)\left(\frac{1}{1.00\,\text{m}} - \frac{1}{-1.00\,\text{m}}\right)\right]^{-1} = 0.86\,\text{m}\,.$$

24.10

It is clear that $f = 2.50\,\text{m}$ for the lens, since $s_i = 2.50\,\text{m}$ when $s_o = \infty$. Now set $s_o = 4.00\,\text{m}$ and find s_i from Eq. (24.6), $1/s_o + 1/s_i = 1/f$:

$$s_i = \frac{s_o f}{s_o - f} = \frac{(4.00\,\text{m})(2.50\,\text{m})}{4.00\,\text{m} - 2.50\,\text{m}} = 6.67\,\text{m}\,,$$

i.e., the screen must be positioned at 6.67 m behind the lens. Since $s_i > 0$ the image is real.

24.16

Use the Eq. (24.6), the Gaussian Lens Equation, with $s_o = 30.0\,\text{cm}$ and $f = +10.0\,\text{cm}$, to find s_i: $1/f = 1/s_o + 1/s_i$, or

$$s_i = \frac{s_o f}{s_o - f} = \frac{(30.0\,\text{cm})(10.0\,\text{cm})}{30.0\,\text{cm} - 10.0\,\text{cm}} = +15.0\,\text{cm}\,,$$

i.e., the image of the cat is located 15.0 cm behind the lens. It is real (since $s_i > 0$), with a magnification of

$$M_T = -\frac{s_i}{s_o} = -\frac{15.0\,\text{cm}}{30.0\,\text{cm}} = -0.500\,,$$

i.e., it is half-sized and inverted (since $M_T < 0$). Here we used Eq. (24.9) for the transverse magnification.

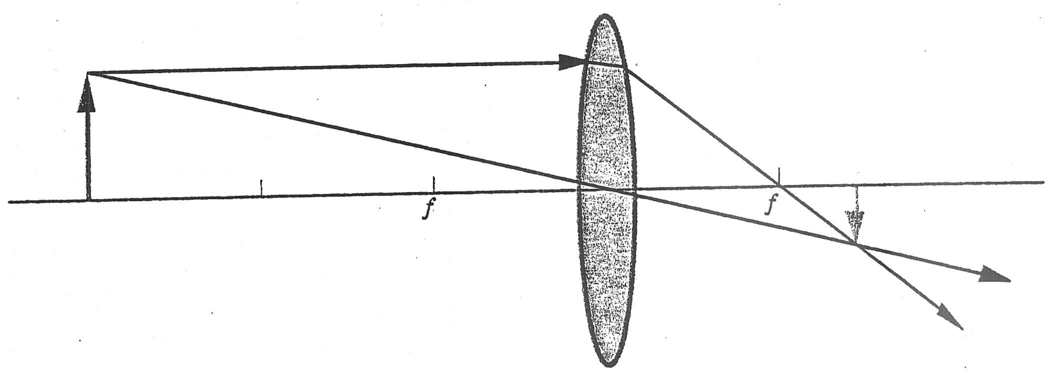
24.21

(a) First, find the location of the image by solving for s_i from Eq. (24.6), which gives

$$s_i = \frac{s_o f}{s_o - f} = \frac{(0.75\,\text{m})(0.25\,\text{m})}{0.75\,\text{m} - 0.25\,\text{m}} = 0.38\,\text{m}\,,$$

so the image is located a distance 0.38 m behind the lens. It is real (since $s_i > 0$), with a magnification of $M_T = -s_i/s_o = -(0.375\,\text{m})/(0.75\,\text{cm}) = -0.50$, so the image is half-sized (since $|M_T| = 0.50$) and inverted (since $M_T < 0$).

(b) The ray diagram is shown below.



24.29

When an object is placed a distance s_o in front of a convex lens of focal length f a real image is formed a distance s_i behind the lens (if $s_o > f$). Here s_o, s_i and f are related via the Gaussian Lens Equation [Eq. (24.6)]: $1/f = 1/s_o + 1/s_i$. In our case the object (grasshopper) is located

10 cm to the left of the lens while its image is 30 cm to the right of the lens, so $s_o = 10\,\text{cm}$ and $s_i = 30\,\text{cm}$. Plug these data into Eq. (24.6) to find f, the focal length of the lens:

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{10\,\text{cm}} + \frac{1}{30\,\text{cm}} = \frac{4}{30\,\text{cm}},$$

and so $f = 30\,\text{cm}/4 = 7.5\,\text{cm}$.

(a) Now the grasshopper jumps 7.5 cm towards the lens so s_o decreases by 7.5 cm, to 10 cm − 7.5 cm = 2.5 cm. Plug this new value of s_o, along with $f = 7.5\,\text{cm}$, into Eq. (24.6) again and solve for s_i, the new location of the image:

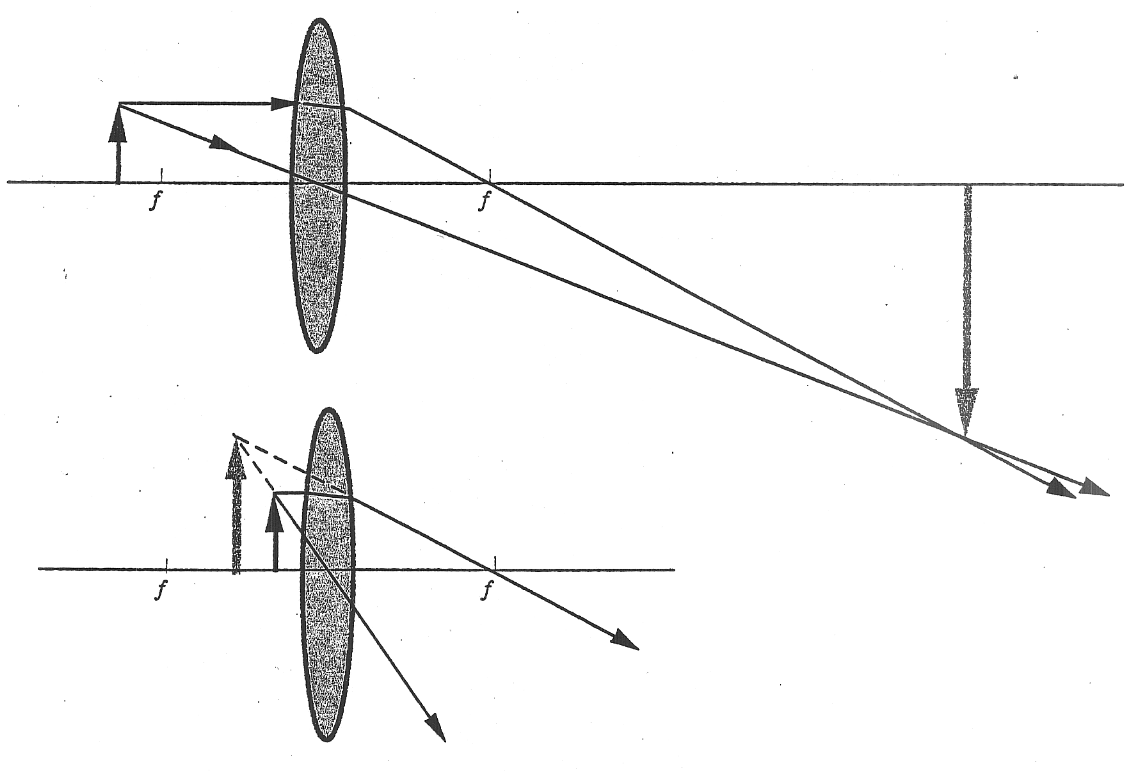
$$s_i = \frac{s_o f}{s_o - f} = \frac{(2.5\,\text{cm})(7.5\,\text{cm})}{2.5\,\text{cm} - 7.5\,\text{cm}} = -3.8\,\text{cm},$$

which means that the image is now 3.8 cm to the left of the lens (i.e., on the same side of the lens as the grasshopper itself), since $s_i < 0$.

(b) At first $s_i = 30\,\text{cm}$ and $s_o = 10\,\text{cm}$, so the transverse magnification is $M_T = -s_i/s_o = -30\,\text{cm}/10\,\text{cm} = -3.0$. The image is real (as $s_i > 0$), inverted (as $M_T < 0$), and magnified to three times the original size of the grasshopper (as $|M_T| = 3.0$).

Similarly, for the new image $s_i = -3.8\,\text{cm} < 0$, so it is now virtual. The transverse magnification is now $M_T = -s_i/s_o = -3.8\,\text{cm}/2.5\,\text{cm} = +1.5 > 0$, so the new image is right-side-up and is magnified to 1.5 times the original size.

(c) The two ray diagrams below depict the situation before and after the grasshopper makes the jump.



24.40

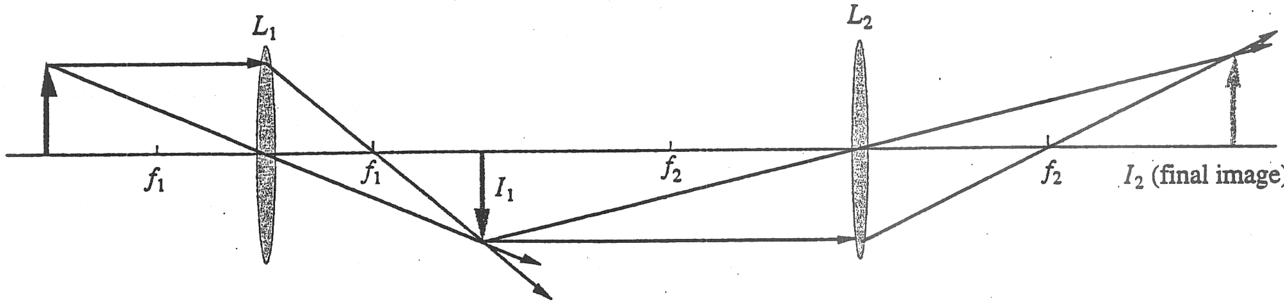
With the previous problem in mind, this time $s_{o1} = 50.0\,\text{cm}$ and $f_1 = 25.0\,\text{cm}$, hence for the first image (I_1)

$$s_{i1} = \frac{s_{o1} f_1}{s_{o1} - f_1} = \frac{(50.0\,\text{cm})(25.0\,\text{cm})}{50.0\,\text{cm} - 25.0\,\text{cm}} = 50.0\,\text{cm}\,,$$

and so $s_{o2} = 130\,\text{cm} - 50.0\,\text{cm} = 80.0\,\text{cm}$ for the second lens, with $f_2 = 40.0\,\text{cm}$. Thus

$$s_{i2} = \frac{s_{o2} f_2}{s_{o2} - f_2} = \frac{(80.0\,\text{cm})(40.0\,\text{cm})}{80.0\,\text{cm} - 40.0\,\text{cm}} = 80.0\,\text{cm}\,,$$

meaning that the final image (I_2) is located a distance 80.0 cm behind the second lens. The ray diagram is shown below.



24.73

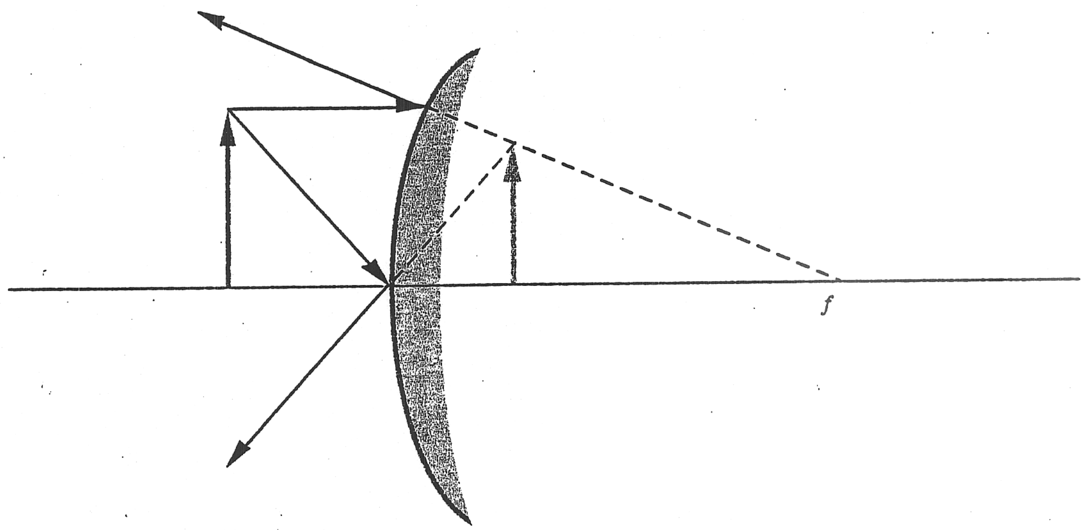
From the hint given in the problem statement, we take $R = +20\,\text{cm}$ and obtain $f = -R/2 = -100\,\text{cm}/2 = -50\,\text{cm}$ for the concave mirror. Plug this value for the focal length, along with $s_o = 20\,\text{cm}$, into $1/s_o + 1/s_i = 1/f$ to find

$$s_i = \frac{s_o f}{s_o - f} = \frac{(20\,\text{cm})(-50\,\text{cm})}{20\,\text{cm} - (-50\,\text{cm})} = -14\,\text{cm}\,,$$

where the negative sign indicates that the image appears to be 14 cm *behind* the vertex of the mirror. It is virtual (as no light from your nose can bypass the mirror and reach behind its vertex), with a transverse magnification of

$$M_T = -\frac{s_i}{s_o} = -\frac{-14.3\,\text{cm}}{20\,\text{cm}} = +0.71\,,$$

so it is right-side-up (as $M_T > 0$) and minified to about 71% the original size of your nose, as shown in the ray diagram in the next page.



24.78

The magnification of the mirror is $y_i/y_o = -s_i/s_o$. Here $y_o = 1.0\,\text{m}$, $y_i = -1.0\,\text{cm}$ (note the negative sign here, since the image is minified and, therefore, real, which means that it must be inverted), and $s_o = 10\,\text{m}$. Thus

$$s_i = -\frac{s_o y_i}{y_o} = -\frac{(10\,\text{m})(-1.0\,\text{cm})}{1.0\,\text{m}} = +10\,\text{cm} = +0.10\,\text{m},$$

so the detector should be located a distance 10 cm away from the vertex of the mirror. Plug this value of s_i into $1/f = 1/s_o + 1/s_i$, along with $s_o = 10\,\text{m}$, to find f, the focal length of the mirror:

$$f = \frac{s_o s_i}{s_o + s_i} = \frac{(10\,\text{m})(0.10\,\text{m})}{10\,\text{m} + 0.10\,\text{m}} = 0.099\,\text{m} = 9.9\,\text{cm}.$$

The ray diagram is shown below.