Solutions to Problems

Chapter 10

10.1

Use Eq. (10.1): $F = ks$. In our case $F = 50\,\mathrm{N}$ and $s = 25\,\mathrm{cm} - 20\,\mathrm{cm} = 5\,\mathrm{cm}$, so the spring constant is

$$k = \frac{F}{s} = \frac{50\,\mathrm{N}}{0.05\,\mathrm{m}} = 1.0\,\mathrm{kN/m}\,.$$

10.2

Again, Use Eq. (10.1): $F = ks$. Here $F = 200\,\mathrm{N}$ and $s = 10.0\,\mathrm{cm} = 0.100\,\mathrm{m}$, so the spring constant is

$$k = \frac{F}{s} = \frac{200\,\mathrm{N}}{0.100\,\mathrm{m}} = 2.00\,\mathrm{kN/m}\,.$$

10.3

The elastic force F provided by the spring is equal to the weight F_w of the monkey: $F_\mathrm{w} = F = ks$. Thus the spring is stretched by

$$s = \frac{F_\mathrm{w}}{k} = \frac{10.0\,\mathrm{N}}{500\,\mathrm{N/m}} = 0.0200\,\mathrm{m} = 2.00\,\mathrm{cm}\,.$$

10.4

Use Eq. (10.1): $F = ks$. Here $F = 10\,\mathrm{N}$ and $k = 500\,\mathrm{N/m}$, so the spring will contract by

$$s = \frac{F}{k} = \frac{10\,\mathrm{N}}{500\,\mathrm{N/m}} = 0.020\,\mathrm{m} = 2.0\,\mathrm{cm}\,.$$

10.5

The work W done on the spring is equal to the gain in its elastic potential energy:

$$W = \Delta\mathrm{PE_e} = \frac{1}{2}ks^2 = \frac{1}{2}(2.00\times10^2\,\mathrm{N/m})(0.100\,\mathrm{m})^2 = 1.00\,\mathrm{J}\,.$$

10.6

Use Eq. (10.1), $F = ks$, where $F = F_\mathrm{w} = mg$ is the weight of the machine of mass m $(= 1000\,\mathrm{kg})$, and $s = 0.500\,\mathrm{cm}$. Thus the spring constant is

$$k = \frac{F}{s} = \frac{mg}{s} = \frac{(1000\,\mathrm{kg})(9.81\,\mathrm{m/s^2})}{0.500\times10^{-2}\,\mathrm{m}} = 1.96\times10^6\,\mathrm{N/m}\,.$$

10.7

In Eq. (10.1), plug in $F = 450\,\text{N}$ and $s = 0.700\,\text{m}$ to find k:

$$k = \frac{F}{s} = \frac{450\,\text{N}}{0.700\,\text{m}} = 643\,\text{N/m}\,.$$

10.8

Use Eq. (10.2): $\Delta\text{PE}_\text{e} = \frac{1}{2}ks^2$. Here $k = 50\,\text{N/m}$ and $s = 0.05\,\text{m}$, so

$$\Delta\text{PE}_\text{e} = \frac{1}{2}ks^2 = \frac{1}{2}(50\,\text{N/m})(0.05\,\text{m})^2 = 0.06\,\text{J}\,.$$

10.9

Suppose that a force F applied to the steel produces a bending in the amount of s. Then from Hooke's Law $F = ks$. When the force increases by ΔF, the corresponding increase in s must satisfy $\Delta F = \Delta(ks) = k\Delta s$, which gives $k = \Delta F/\Delta s$. According to the problem statement, for $\Delta F = 2.00\,\text{N}$ we have $\Delta s = 1.00\,\text{cm} = 0.0100\,\text{m}$; and so $k = \Delta F/\Delta s = 2.00\,\text{N}/0.0100\,\text{m} = 200\,\text{N/m}$. It follows from Eq. (10.2) that the work W needed to bend the steel by an amount s $(= 2.50\,\text{cm})$, or the change in the *elastic*-PE of the steel, is

$$W = \Delta\text{PE}_\text{e} = \frac{1}{2}ks^2 = \frac{1}{2}(200\,\text{N/m})(0.0250\,\text{m})^2 = 0.0625\,\text{J} = 62.5\,\text{mJ}\,.$$

10.10

The force F that needs to be applied to a spring to stretch its length by an amount x is $F = kx$, where k is its spring constant. In our case a 40.0-N force results in a stretch of $14.0\,\text{cm} - 10.0\,\text{cm} = 4.0\,\text{cm} = 0.040\,\text{m}$, and so $k = 40.0\,\text{N}/0.040\,\text{m} = 1.0 \times 10^3\,\text{N/m}$. Thus the work W that must be done to stretch the spring from $x_\text{i} = 14.0\,\text{cm} - 10.0\,\text{cm} = 0.40\,\text{cm}$ to $x_\text{f} = 18.0\,\text{cm} - 10.0\,\text{cm} = 8.0\,\text{cm}$ is

$$\begin{aligned} W &= \int dW = \int_{x_\text{i}}^{x_\text{f}} F(x)\,dx = \int_{x_\text{i}}^{x_\text{f}} kx\,dx = \frac{1}{2}k\left(x_\text{f}^2 - x_\text{i}^2\right) \\ &= \frac{1}{2}(1.0 \times 10^3\,\text{N/m})\left[(0.080\,\text{m})^2 - (0.040\,\text{m})^2\right] \\ &= 2.4\,\text{J}\,. \end{aligned}$$

10.11

Using Eq. (10.1), we find that the spring will stretch by $s = F/k$ under a load of F. When the load changes to F', the new amount of stretch of the spring is $s' = F'/k$. Thus $s' - s =$

the motion, which coincides with the center of the record, is R, the radius of the record. Since the record turns at 78 rpm, the period of the motion is $T = (1/78)$ min, and the corresponding frequency is

$$f = \frac{1}{T} = \frac{78}{(1\text{min})(60\,\text{s/min})} = 1.3\,\text{Hz}.$$

According to Eq. (10.12) the angular frequency is $\omega = 2\pi f = (2\pi\,\text{rad})(1.3\,\text{s}^{-1}) = 8.2\,\text{rad/s}$.

10.62

In each cycle of its up-and-down motion the insect falls freely through a distance $y = 0.20$ mm, which takes a time t satisfying $y = \frac{1}{2}gt^2$. Solve for t:

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(2.0 \times 10^{-4}\,\text{m})}{9.81\,\text{m/s}^2}} = 6.39 \times 10^{-3}\,\text{s}.$$

Suppose that the insect spends the same amount of time per cycle flying upward and falling downward, then the time it takes for it to complete a whole cycle, or the period T, is $T = 2t = 2(6.39 \times 10^{-3}\,\text{s}) = 0.13\,\text{s}$. The corresponding frequency is $f = 1/T = 1/0.128\,\text{s} = 78\,\text{Hz}$.

10.63

The acceleration as a function of time of an object in SHM is given by Eq. (10.17): $a_x = -A\omega^2 \cos\omega t$. For maximum acceleration $a_x(\text{max})$, set $\cos\omega t = \pm 1$ to obtain $|a_x(\text{max})| = A\omega^2 = A(2\pi f)^2$. With $A = 0.50$ cm and $f = 50$ Hz,

$$|a_x(\text{max})| = 4\pi^2 A f^2 = 4\pi^2(0.50 \times 10^{-2}\,\text{m})(50\,\text{Hz})^2 = 4.9 \times 10^2\,\text{m/s}^2.$$

10.64

(a) The velocity of an SHM as a function of time is given by Eq. (10.15), $v_x = -A\omega \sin\omega t$. Drop the minus sign (since we only need the speed) and set $\sin\omega t = 1$ to obtain the speed at the equilibrium position (where the magnitude of v_x reaches its maximum value):

$$|v_x(\text{max})| = v_{\text{max}} = A\omega = A(2\pi f) = 2\pi(0.020\,\text{m})(10\,\text{Hz}) = 1.3\,\text{m/s}.$$

(b) Use Eq. (10.18): $a_x = -\omega^2 x$. At maximum displacement $x = \pm A$, where $a = -\omega^2(\pm A) = \mp A\omega^2$. Drop the minus sign to obtain the magnitude of the acceleration to be

$$|a| = A\omega^2 = A(2\pi f)^2 = 4\pi^2(0.020\,\text{m})(10\,\text{Hz})^2 = 79\,\text{m/s}^2,$$

which is about 8.1 times g, the acceleration of gravity.

10.65

Use Eq. (10.18), $a = -\omega^2 x = -4\pi^2 f^2 x$. Plug in $a = +0.40\,\mathrm{m/s^2}$ and $x = -20\,\mathrm{cm}$ (where the minus sign is necessary since we have set a to be positive; otherwise f^2 would be negative, which is impossible), and solve for $T = 1/f$:

$$T = \frac{1}{f} = 2\pi\sqrt{-\frac{x}{a}} = 2\pi\sqrt{-\frac{-0.20\,\mathrm{m}}{0.40\,\mathrm{m/s^2}}} = 4.4\,\mathrm{s}\,.$$

10.66

The speed of an object in SHM as it passes through the equilibrium point is its maximum value, given by $v_{max} = A\omega = A(2\pi f)$. Plug in $A = 0.50\,\mathrm{mm}$ and $v_0 = 1.57\,\mathrm{m/s}$ and solve for f:

$$f = \frac{v_{max}}{2\pi A} = \frac{1.57\,\mathrm{m/s}}{2\pi(0.50\times 10^{-3}\,\mathrm{m})} = 0.50\,\mathrm{kHz}\,.$$

10.67

Compare the expression $x = 5.0\cos(0.40t + 0.10)$ for the SHM of the body in question with Eq. (10.13), $x = A\cos\omega t = x_{max}\cos 2\pi f t$, which describes a standard SHM with an initial displacement of A.

(a) $A = 5.0\,\mathrm{m}$, by direct comparison between the two equations.

(b) Set $2\pi f = 0.40\,\mathrm{s^{-1}}$ to obtain $f = 0.40\,\mathrm{s^{-1}}/2\pi = 0.064\,\mathrm{Hz}$.

(c) Set $t = 0$ in the expression for the phase to obtain the initial phase: $\varepsilon = 0.40\times 0 + 0.10 = 0.10\,\mathrm{rad}$.

(d) Plug $t = 2.0\,\mathrm{s}$ into the expression for x: $x = 5.0\cos\left[(0.40\,\mathrm{s^{-1}})(2.0\,\mathrm{s}) + 0.10\right] = 3.1\,\mathrm{m}$.

10.68

Compare the expression $x = 8.0\cos(1.2t + 0.4)$ for the SHM of the body in question with Eq. (10.13): $x = x_{max}\cos\omega t$. This gives $\omega = 1.2\,\mathrm{rad/s}$, so

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.2\,\mathrm{rad/s}} = 5.2\,\mathrm{s}\,.$$

10.69

Since $d\cos(\omega t)/dt = -\omega\sin\omega t$,

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(x_{max}\cos 2\pi f t) = -x_{max}(2\pi f)\sin 2\pi f t\,.$$

where in the last expression v_x is measured in m/s and t is in seconds.

10.77
Similar to the previous problem, the general expression for the displacement z of the particle as a function of time is $z = A\cos(2\pi ft + \varepsilon)$, where $A = 0.50\,\text{m}$ and $f = 0.20\,\text{Hz}$. At $t = 0$ we have $z = A\cos(0 + \varepsilon) = +0.50\,\text{m}$, so $\cos\varepsilon = +0.50\,\text{m}/0.50\,\text{m} = +1$, or $\varepsilon = 0$. Thus

$$z = A\cos(2\pi ft + \varepsilon) = (0.50\,\text{m})\cos[2\pi(0.20\,\text{Hz})t] = (0.50\,\text{m})\cos(0.40\pi t),$$

where t is in seconds. Plug $t = 5.00\,\text{s}$ into this expression to obtain $z = (0.50\,\text{m})\cos(2\pi) = +0.50\,\text{m}$. Similarly, you can find the displacement values at $t = 2.50\,\text{s}$ and $1.25\,\text{s}$ to be $z = -0.50\,\text{m}$ and $z = 0$, respectively.

10.78
When the displacement of the mass is x and its speed is v_x, we have $\text{PE} = \frac{1}{2}kx^2$ and $\text{KE} = \frac{1}{2}mv_x^2$. Since $x = A\cos\omega t$ and $v_x = dx/dt = d(A\cos\omega t)/dt = -\omega A\sin\omega t$,

$$\text{E} = \text{KE} + \text{PE} = \frac{1}{2}kx^2 + \frac{1}{2}mv_x^2 = \frac{1}{2}k(A\cos\omega t)^2 + \frac{1}{2}m(-\omega A\sin\omega t)^2,$$

where the KE term may be rewritten, upon using $\omega^2 = k/m$, as

$$\text{KE} = \frac{1}{2}m\omega^2A^2\sin^2\omega t = \frac{1}{2}m\left(\frac{k}{m}\right)A^2\sin^2\omega t = \frac{1}{2}kA^2\sin^2\omega t.$$

Substitute this expression for KE into that for E:

$$\text{E} = \frac{1}{2}kA^2\cos^2\omega t + \frac{1}{2}kA^2\sin^2\omega t = \frac{1}{2}kA^2\left(\sin^2\omega t + \cos^2\omega t\right) = \frac{1}{2}kA^2,$$

where in the last step the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ was used, with $\theta = \omega t$. E is therefore a constant independent of t.

10.79
The mechanical energy E of a spring-mass system consists of the kinetic energy KE of the mass m, given by $\text{KE} = \frac{1}{2}mv_x^2$, where $v_x = dx/dt$ is the speed of the mass; and the *elastic*-PE of the spring as it is stretched or compressed by an amount x: $\text{PE}_e = \frac{1}{2}kx^2$, with k the spring constant:

$$\text{E} = \text{KE} + \text{PE}_e = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2.$$

To observe the Conservation of Energy, set $d\text{E}/dt = 0$. The derivative for the KE term yields

$$\frac{d(\text{KE})}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv_x^2\right) = \frac{1}{2}m\left(\frac{dv_x^2}{dt}\right) = \frac{1}{2}m\left(2v_x\frac{dv_x}{dt}\right) = mv_x\left(\frac{dv_x}{dt}\right) = mv_x\left(\frac{d^2x}{dt^2}\right),$$