The astronaut is stationary relative to himself, so the time he measures for his own blood circulation is $\Delta t_s = 45\,\mathrm{s}$. To an Earth-based observer, the corresponding time interval is Δt_M, with $\gamma = 2$; so from Eq. (26.2)

$$\Delta t_M = \gamma \Delta t_s = 2(45\,\mathrm{s}) = 90\,\mathrm{s}.$$

26.7

In this case the life time of the particle measured in the lab frame is $\Delta t_M = 20\,\mathrm{ns}$, and Δt_s, the life time measured when the particle is stationary, satisfies $\Delta t_M = \gamma \Delta t_s$; so

$$\Delta t_s = \frac{\Delta t_M}{\gamma} = \frac{20\,\mathrm{ns}}{10} = 2.0\,\mathrm{ns}.$$

26.10

The one-hour time interval the astronaut sets for her nap is measured onboard the spaceship, in which the event (the nap) is stationary. So $\Delta t_s = 1.00\,\mathrm{h}$. For an observer on Earth who is moving relative to the spaceship at $V = 0.600c$, the nap should last Δt_M, with

$$\Delta t_M = \frac{\Delta t_s}{\sqrt{1 - V^2/c^2}} = \frac{1.00\,\mathrm{h}}{\sqrt{1 - (0.600c/c)^2}} = 1.25\,\mathrm{h},$$

which is how long the flight controller on Earth should let her sleep as measured on his clock.

26.12

The time dilation factor is $\Delta t_M / \Delta t_s = \gamma = 1/\sqrt{1 - \beta^2}$, where $\beta = V/c$, with $V = 1800\,\mathrm{mi/h}$. Convert the unit of V into m/s: $V = (1800\,\mathrm{mi/h})(1609\,\mathrm{m/mi})(1.000\,\mathrm{h}/3600\,\mathrm{s}) = 804.67\,\mathrm{m/s}$. Thus $\beta = V/c = (804.67\,\mathrm{m/s})/(2.998 \times 10^8\,\mathrm{m/s}) = 2.684 \times 10^{-6} \ll 1$, so from the binomial approximation $(1 + x)^n \approx 1 + nx$ for $|x| \ll 1$, we have, with $n = -\frac{1}{2}$ and $x = -\beta^2$,

$$\gamma = (1 - \beta^2)^{-\frac{1}{2}} \approx 1 + \left(-\frac{1}{2}\right)(-\beta^2) = 1 + \frac{1}{2}(2.684 \times 10^{-6})^2 = 1.000\,000\,000\,003\,6,$$

which equals 1.000 to four significant figures — which is as many as we can keep. If the approximation is not used, then

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{1 - (2.684 \times 10^{-6})^2}},$$

26.14

The life time of the particle, measured in its rest frame, is $\Delta t_s = 20\,\mathrm{ns}$. When it travels at $V = 0.80c$ with respect to the laboratory its life time measured in the laboratory frame is now dilated to $\Delta t_M = \Delta t_s / \sqrt{1 - V^2/c^2}$. If the particle moves at a uniform speed of $V = 0.8c$ during its entire life time, it covers a distance of

$$L = v\Delta t_M = \frac{v\Delta t_s}{\sqrt{1 - (V/c)^2}} = \frac{(0.8 \times 2.998 \times 10^8\,\mathrm{m/s})(20 \times 10^{-9}\,\mathrm{s})}{\sqrt{1 - (0.8c/c)^2}} = 8\,\mathrm{m}.$$